

Stackelberg equilibrium in oligopoly: An explanation based on managerial incentives

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Abstract

This paper shows that in a model of managerial delegation in a duopoly, if an owner's decision to hire a manager is modeled explicitly, then the subgame perfect equilibrium may coincide with the Stackelberg solution. Interestingly, this can happen even when the cost of hiring a manager is the same for the owner of each firm.

Keywords: Oligopoly; Managerial incentive; Stackelberg solution; Principal–agent

JEL classification: D21; L13; L20

1. Introduction

The aim of this paper is to suggest a new explanation for Stackelberg leadership in a duopoly. There is a growing literature that theorizes about why there may be a Stackelberg outcome in some duopolies and which firm will be the leader (see, for example, Gale-Or, 1985, and Boyer and Moreaux, 1987). The present paper shows that if owners of firms are free to delegate output decisions to managers appointed by them, then in the subgame perfect equilibrium a duopoly may equilibrate at a Stackelberg outcome. I proceed by adapting the models of Vickers (1984), Fershtman and Judd (1987) and Sklivas (1987) so as to allow owners of firms a larger and more realistic menu of choices.

In Fershtman and Judd, and Sklivas, attention is restricted to a class of linear contracts that owners may offer to the managers. After the owners have done so, each manager decides how much to produce. In these papers, however, the employer's decision to hire or not hire a manager is not explicitly modeled. In effect, the existing models look at the case in which the managers are already installed in the firms.

If, however, the decision to hire a manager is endogenous, then these models yield a very interesting possibility. The purpose of this paper is to demonstrate that for a class of parameters, the subgame perfect equilibrium strategies lead to production levels that are

exactly equal to what would happen under Stackelberg leadership. We then identify the conditions under which a particular firm becomes a Stackelberg leader.

2. The model

In this section a related version of the Vickers, Sklivas, and Fershtman and Judd model is outlined. A duopoly faces the following inverse demand function:

$$p = a - b(x_1 + x_2),$$

where $a, b > 0$, p is price and x_i is firm i 's output. Firm i 's cost of producing x_i units is given by $c_i x_i$. As usual, firm i 's profit function and sales function are given by, respectively,

$$\pi_i = \pi_i(x_1, x_2) = (a - b(x_1 + x_2) - c_i)x_i, \quad (1)$$

and

$$S_i = S_i(x_1, x_2) = (a - b(x_1 + x_2))x_i. \quad (2)$$

In order to make the hiring decision explicit let us suppose that in period 1 the *owners* of firms decide whether to hire *managers* or not. In particular, each owner i selects $m_i \in \{0, 1\}$, where $m_i = 0$ means owner i does not hire a manager and $m_i = 1$ means i does hire a manager. Once a manager is chosen and the manager's objective function specified, which happens in period 2, the manager decides how much to produce (in period 3). In the absence of a manager the decision (in period 3) is taken by the owner.

In period 2 each owner (who has a manager) picks an objective function for the manager. Manager i 's objective function can only belong to the following class.¹

$$R_i = R_i(\alpha_i, x_1, x_2) \equiv \alpha_i \pi_i + (1 - \alpha_i) S_i. \quad (3)$$

In other words, owners 1 and 2 select α_1 and α_2 , respectively, in period 2. Actually, manager i is told that her salary is

$$A_i + B_i R_i,$$

where A_i and B_i are constants. Clearly, maximizing $A_i + B_i R_i$ and maximizing (3) are equivalent if the control variable is x_i . Hence, when speaking of managerial behavior we shall speak as if the manager's objective function were $R_i(\alpha_i, x_1, x_2)$.

A_i and B_i are chosen by owner i to simply ensure that the participation constraint is satisfied; that is, $A_i + B_i R_i$ in equilibrium happens to be equal to the manager's reservation income. Let us suppose that manager i 's reservation income is Y_i . In addition, assume that the owner finds that he can get away from the firm for several hours once he has a manager. Let

¹ See, also, D'Aspremont and Gerard-Varet (1980), Fershtman (1985) and Katz (1986).

X_i be the amount that owner i can earn elsewhere in the time that gets released in this manner. Hence owner's i 's cost of hiring a manager is

$$Z_i = Y_i - X_i. \quad (4)$$

Owner i 's *net* profit, if he hires a manager, will be $\pi_i - Z_i$.

It seems reasonable to assume $Z_i > 0$. This is because an owner cannot put all his time in another trade even if he has a manager. He will still have to do *some* supervision. What is interesting and at first sight counter-intuitive is that Stackelberg leadership can arise even when $Z_i = Z_j$.

The model of Fershtman–Judd and Sklivas is a special case of the above game; namely, one in which in period 1, $(m_1, m_2) = (1, 1)$ is given. The subgame that occurs after this coincides exactly with their model and the subgame perfect equilibrium of this *subgame* will be referred to as an 'incentive equilibrium'. The values taken by α_i , x_i and π_i in such a subgame perfect equilibrium are given by

$$\bar{\alpha}_i = (8c_i - a - 2c_j)/5c_i, \quad j \neq i, \quad (5)$$

$$\bar{x}_i = (2a - 6c_i + 4c_j)/5b, \quad j \neq i, \quad (6)$$

$$\bar{\pi}_i = 2(a - 3c_i + 2c_j)^2/25b. \quad (7)$$

It follows from (3) that

$$x_i = \underset{x_i}{\operatorname{argmax}} R_i(\alpha_i, x_1, x_2) = (a - bx_2 - \alpha_1 c_1)/2b. \quad (8)$$

I shall refer to (8) as the *managerial reaction function* of firm i . The owner's reaction function, referred to here as simply the *reaction function*, is clearly the special case of (8) with $\alpha_i = 1$. By solving the two equations in (8), with $i = 1, 2$, we get $x_1(\alpha_1, \alpha_2)$ and $x_2(\alpha_1, \alpha_2)$. By inserting these in (1) we get π_i as a function of (α_1, α_2) . By treating this as the pay-off function of a normal-form game (where α_1 and α_2 are the strategic variables), and solving for the Nash equilibrium, we get $\bar{\alpha}_1$ and $\bar{\alpha}_2$. Then $\bar{\pi}_i$ in (7) is derived by replacing x_1 and x_2 with $x_1(\bar{\alpha}_1, \bar{\alpha}_2)$ and $x_2(\bar{\alpha}_1, \bar{\alpha}_2)$.

3. The Stackelberg solution

The subgame perfect equilibrium of the full game described in Section 2 – namely, a game in which the hiring decision is an explicit one – however, need not coincide with the incentive equilibrium described in (5)–(7). For a certain class of parameters it would, in fact, coincide with the standard, textbook Stackelberg equilibrium. In most conventional treatments, a Stackelberg solution is either *assumed* or directly deduced from an *exogenously* imposed temporal structure of moves. At times it is *motivated* by pre-entry configurations (as in, for example, Basu and Singh, 1990) but it is left at the level of motivation. In the present model, the occurrence of Stackelberg is more endogenous, and who will be leader depends on the cost parameters of the model.

To see this, let us first check what happens after each possible one-period history. We have already seen what happens after $(m_1, m_2) = (1, 1)$. Next, consider the case $(m_1, m_2) = (0, 0)$; that is, no one hires a manager. This gives us the standard Cournot case and as is easily worked out or, even more easily, lifted from some microeconomics textbook, firm i 's profit in the Cournot equilibrium is given by

$$\pi_{iN} = (a - 2c_i + c_j)^2 / 9b. \quad (9)$$

Finally, and without loss of generality, consider the history $(m_1, m_2) = (0, 1)$. This case is solved in Fershtman (1985). Since $m_1 = 0$, firm 1's reaction in period 3 is the owner's reaction function, given by setting $i = 1$ and $\alpha_1 = 1$ in (8). This is described by AB in Fig. 1.

Let CD be owner 2's reaction function. Since, by choosing α_2 , owner 2 can make any line parallel to CD firm 2's managerial reaction function, it is obvious that in period 2 owner 2 would choose α_2 such that the managerial reaction function is $C''D''$, which is a line that goes through point S , which is standard Stackelberg outcome with firm 2 as leader. The line π_2'' is firm 2's iso-profit curve.

The profit earned by firm 1 at point S in Fig. 1 is denoted by π_{1F} . The subscript is explained by the fact that firm 1 is a 'follower' at point S . The computation of π_{1F} is standard:

$$\pi_{1F} = (a - 3c_1 + 2c_2)^2 / 16b, \quad (10)$$

π_{2F} is symmetric.

If π_{1L} is 1's profit when 1 is Stackelberg leader, it is easy to check that

$$\pi_{1L} = (a - 2c_1 + c_2)^2 / 8b. \quad (11)$$

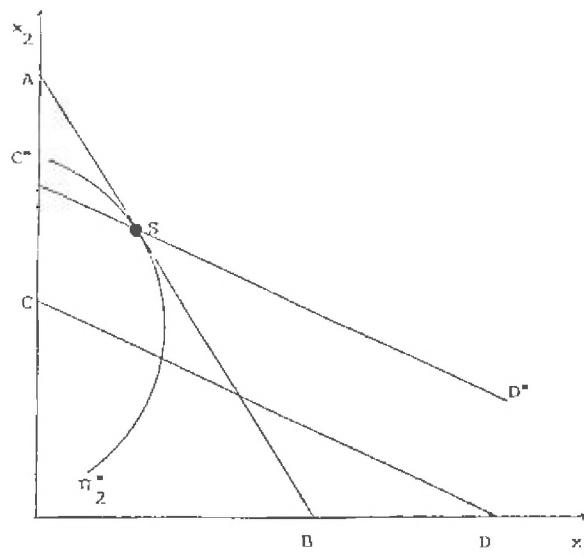


Fig. 1.

Recalling that hiring a manager costs firm 1, Z_1 , owner 1's pay-offs that occur in the subgame perfect equilibria of the subgames that occur after the four possible period-1 histories, $(0, 0)$, $(1, 1)$, $(0, 1)$, $(1, 0)$, are summarized in the following partial pay-off matrix:

| | | Owner 2 | |
|---------|---|----------------------------|--|
| | | 0 | 1 |
| Owner 1 | 0 | π_{1N}, π_{2N} | $\pi_{1F}, \pi_{2L} - Z_2$ |
| | 1 | $\pi_{1L} - Z_1, \pi_{2F}$ | $\tilde{\pi}_1 - Z_1, \bar{\pi}_2 - Z_2$ |

The incentive equilibrium is a subgame perfect equilibrium of the full game only if $\pi_1 - Z_1 \geq \pi_{1F}$ and $\bar{\pi}_2 - Z_2 \geq \pi_{2F}$. It is easy to work out these conditions in terms of the exogenous parameters of the model. If, on the other hand,

$$\pi_{1F} > \bar{\pi}_1 - Z_1 \text{ or, equivalently, } Z_1 > \frac{7(a - 3c_1 + 2c_2)^2}{400b}, \quad (12)$$

and

$$\pi_{2L} - Z_2 > \pi_{2N} \text{ or, equivalently, } \frac{(a - 2c_2 + c_1)^2}{72b} > Z_2, \quad (13)$$

then a subgame perfect equilibrium of this model coincides with the standard Stackelberg outcome with 2 as leader.

From (12) and (13) it follows that the equilibrium is likely to coincide with the Stackelberg outcome with firm 2 as leader if (i) firm 1's marginal cost of production is high, relative to firm 2's marginal cost and (ii) Z_1 is high relative to Z_2 .

Since Z_1 and Z_2 are the costs of hiring a manager, it may at first sight appear that the Stackelberg result is being driven by (ii). However, surprisingly, even if $Z_1 = Z_2 = Z$, (12) and (13) can be satisfied. All we need is that

$$\frac{7(a - 3c_1 + 2c_2)^2}{400b} < Z < \frac{(a - 2c_2 + c_1)^2}{72b}. \quad (14)$$

And this can be true if c_1 is sufficiently high compared with c_2 . Note, however, that if $Z = 0$, then (14) cannot be true and, therefore, that Stackelberg cannot occur.

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